

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

--	--	--	--	--

--	--	--	--

## Pearson Edexcel International GCSE

Time 2 hours

Paper  
reference

**4PM1/01**

### Further Pure Mathematics PAPER 1



Calculators may be used.

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

P71661A

©2022 Pearson Education Ltd.

Q:1/1/1/



Pearson

## International GCSE in Further Pure Mathematics Formulae sheet

### Mensuration

Surface area of sphere =  $4\pi r^2$

Curved surface area of cone =  $\pi r \times$  slant height

Volume of sphere =  $\frac{4}{3}\pi r^3$

### Series

#### Arithmetic series

Sum to  $n$  terms,  $S_n = \frac{n}{2}[2a + (n - 1)d]$

#### Geometric series

Sum to  $n$  terms,  $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity,  $S_\infty = \frac{a}{1 - r} \quad |r| < 1$

#### Binomial series

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

### Calculus

#### Quotient rule (differentiation)

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

### Trigonometry

#### Cosine rule

In triangle  $ABC$ :  $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

### Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Given that  $\frac{2\sqrt{3} - 4}{3\sqrt{3} + 5}$  can be written in the form  $a + b\sqrt{3}$  where  $a$  and  $b$  are integers,

find, without using a calculator, the value of  $a$  and the value of  $b$

Show your working clearly.

(3)

Area with horizontal dotted lines for working.

(Total for Question 1 is 3 marks)



P 7 1 6 6 1 A 0 3 3 2

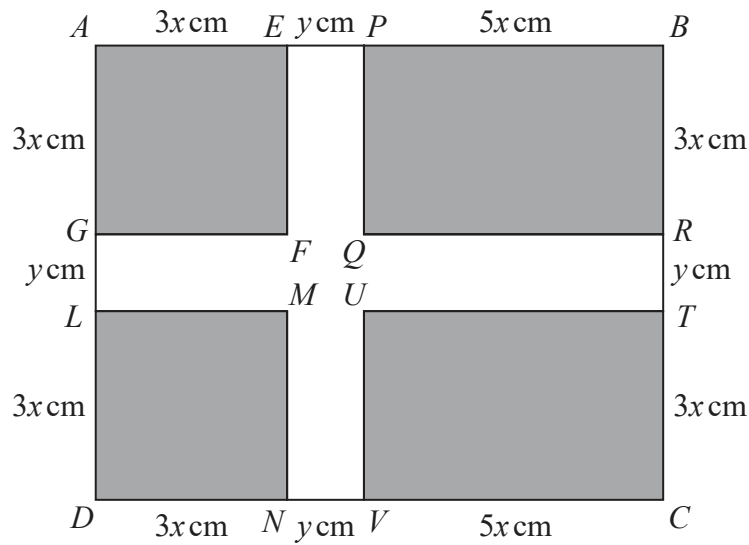


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the design for a flag consisting of a white cross on a grey background.

$AEFG$  and  $DLMN$  are squares with sides of length  $3x$  cm.

$BPQR$  and  $CTUV$  are rectangles with sides of length  $5x$  cm and  $3x$  cm.

The width of the cross is  $y$  cm.

The total area of the flag is  $H$  cm<sup>2</sup>

(a) Write down an expression, in terms of  $x$  and  $y$ , for  $H$ .

(1)

Given that the area of the cross is  $K$  cm<sup>2</sup>

(b) show that  $K = 14xy + y^2$

(3)

The total area of the flag is to be 3432 cm<sup>2</sup> and the area of the cross is to be 1080 cm<sup>2</sup>

(c) Find the value of  $x$  and the value of  $y$

(5)

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 2 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 25 horizontal dotted lines.



P 7 1 6 6 1 A 0 5 3 2

**Question 2 continued**

Area with horizontal dotted lines for writing.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 2 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 2 is 9 marks)**



P 7 1 6 6 1 A 0 7 3 2

3

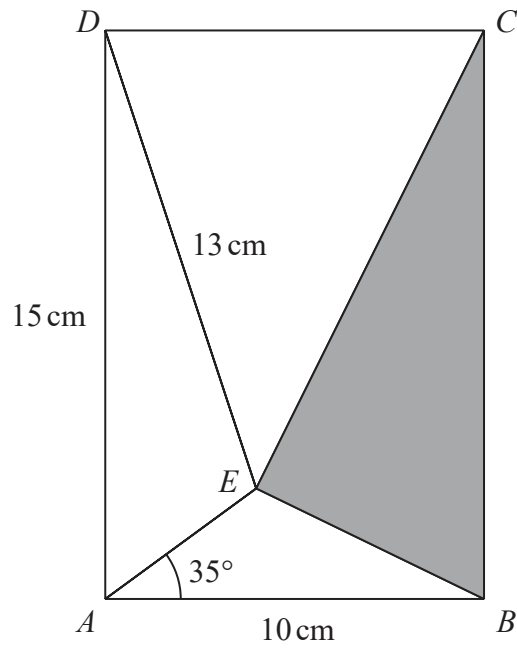


Diagram NOT accurately drawn

Figure 2

Figure 2 shows a rectangle  $ABCD$  with  $AB = 10$  cm and  $AD = 15$  cm.  
 $E$  is the point inside the rectangle such that  $DE = 13$  cm and angle  $BAE = 35^\circ$

Given that angle  $AED$  is obtuse,

find the area, in  $\text{cm}^2$  to one decimal place, of triangle  $BCE$ .

(7)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA





**Question 3 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 28 horizontal dotted lines.

**(Total for Question 3 is 7 marks)**



4 The common ratio of a geometric series  $G$  is positive.

The sum of the first 4 terms of  $G$  is 80

The sum to infinity of  $G$  is 81

Show that the sum of the first 7 terms of  $G$  differs from the sum to infinity of  $G$  by  $\frac{1}{27}$

(7)

A series of horizontal dotted lines for writing the solution.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 4 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 4 is 7 marks)**



P 7 1 6 6 1 A 0 1 1 3 2

5 Given that  $(2 + 3x)^{-1}$  can be expressed in the form  $p(1 + qx)^{-1}$  where  $p$  and  $q$  are constants,

(a) find the value of  $p$  and the value of  $q$  (2)

(b) Hence expand  $(2 + 3x)^{-1}$  in ascending powers of  $x$  up to and including the term in  $x^3$ , expressing each coefficient as an exact fraction in its lowest terms. (3)

$$f(x) = \frac{1 + x}{2 + 3x}$$

(c) Obtain a series expansion for  $f(x)$ , in ascending powers of  $x$  up to and including the term in  $x^3$ , expressing each coefficient as an exact fraction in its lowest terms. (2)

(d) Hence use algebraic integration to obtain an estimate, to 4 decimal places, of

$$\int_0^{0.5} f(x) \, dx \quad (4)$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



**Question 5 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 25 horizontal dotted lines.



P 7 1 6 6 1 A 0 1 3 3 2

**Question 5 continued**

Handwriting practice area consisting of 25 horizontal dotted lines.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 5 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 5 is 11 marks)**



P 7 1 6 6 1 A 0 1 5 3 2

6 (a) Complete the table of values for  $y = 1 + 3e^{-x}$  giving your answers to 2 decimal places where appropriate.

(2)

$x$	0	0.25	0.5	1	1.5	2	3
$y$		3.34	2.82		1.67		1.15

(b) On the grid opposite, draw the graph of  $y = 1 + 3e^{-x}$  for  $0 \leq x \leq 3$

(2)

(c) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$x = e^{-x} \quad \text{in the interval } 0 \leq x \leq 3$$

(3)

(d) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$\ln(x - 1)^3 = -3x \quad \text{in the interval } 0 \leq x \leq 3$$

(4)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

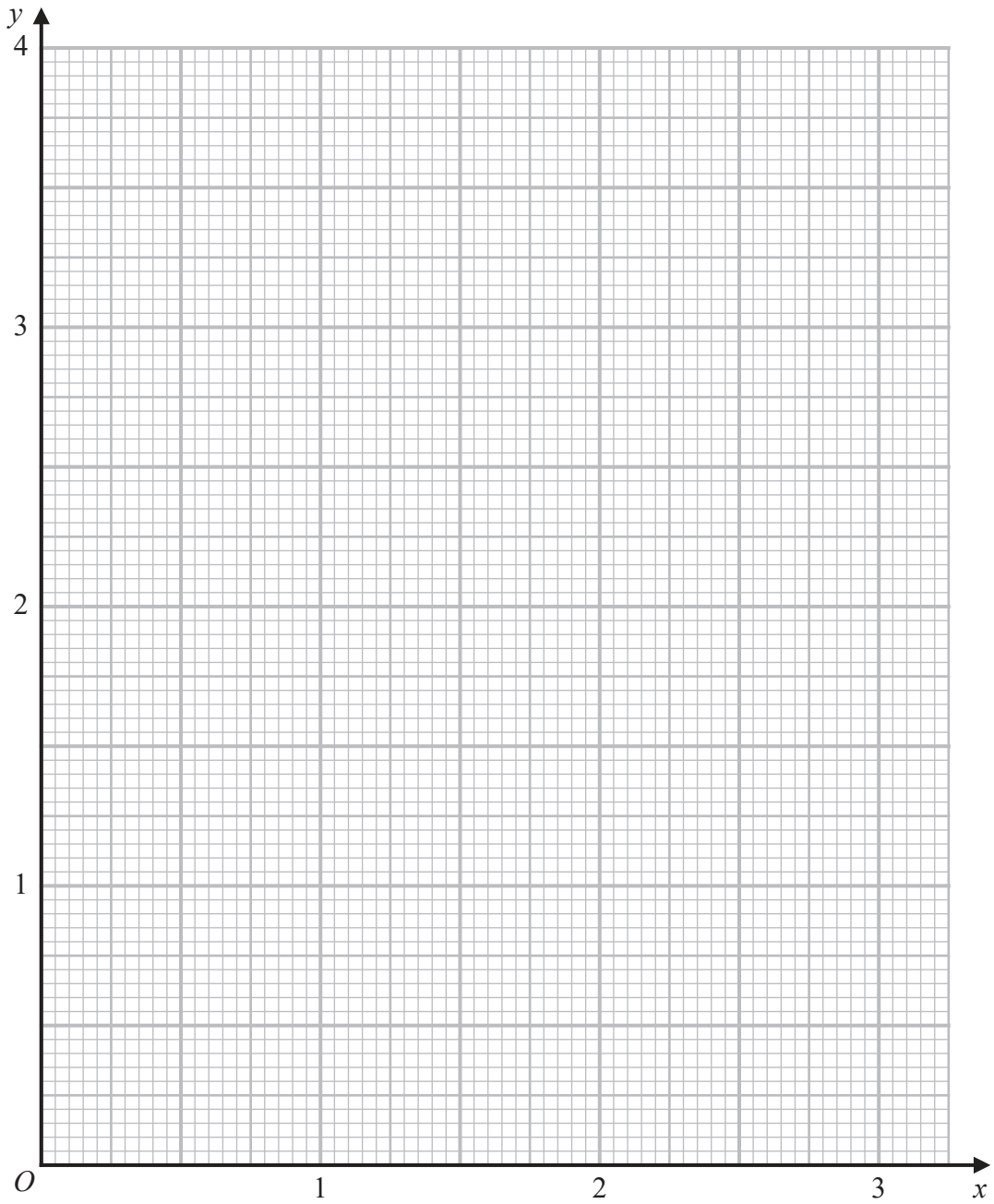
.....

.....





Question 6 continued



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

.....

.....

.....

.....

.....

.....

Turn over for a spare grid if you need to redraw your graph.



P 7 1 6 6 1 A 0 1 7 3 2

**Question 6 continued**

Handwriting practice area consisting of 25 horizontal dotted lines.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 6 continued

Only use this grid if you need to redraw your graph.



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

.....

.....

.....

.....

.....

(Total for Question 6 is 11 marks)



7 The point with coordinates  $(4, -104)$  lies on the curve  $C$  with equation  $y = f(x)$

Given that  $f'(x) = 4x^3 - 12x^2 - 19x + 12$

(a) (i) show that  $C$  passes through the origin, (4)

(ii) show that  $C$  has a maximum at the point on the curve where  $x = 0.5$  (3)

The curve  $C$  has another turning point at  $A$  and another turning point at  $B$ .

Given that the  $x$  coordinate of  $A$  is negative,

(b) (i) find the coordinates of  $A$  and the coordinates of  $B$ , (5)

(ii) determine the nature of these turning points. (3)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



**Question 7 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 25 horizontal dotted lines.



P 7 1 6 6 1 A 0 2 1 3 2

**Question 7 continued**

Area with horizontal dotted lines for writing.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 7 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 7 is 15 marks)**



P 7 1 6 6 1 A 0 2 3 3 2

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

8 The volume of a sphere is  $500 \text{ cm}^3$

(a) Calculate the radius, in cm to 3 significant figures, of the sphere. (2)

The surface area of the sphere is increased by  $20 \text{ cm}^2$

(b) Using calculus, find an estimate for the increase in the radius, in cm to 2 significant figures, of the sphere. (5)

Dotted lines for writing answers.





**Question 8 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area consisting of 28 horizontal dotted lines.

**(Total for Question 8 is 7 marks)**



P 7 1 6 6 1 A 0 2 5 3 2

9

$$f(x) = 3x^4 + 4x^3 - 36x^2 + 64$$

Given that  $f(x)$  can be written in the form  $(x - 2)^2(ax^2 + bx + c)$

(a) find the value of  $a$ , the value of  $b$  and the value of  $c$

(4)

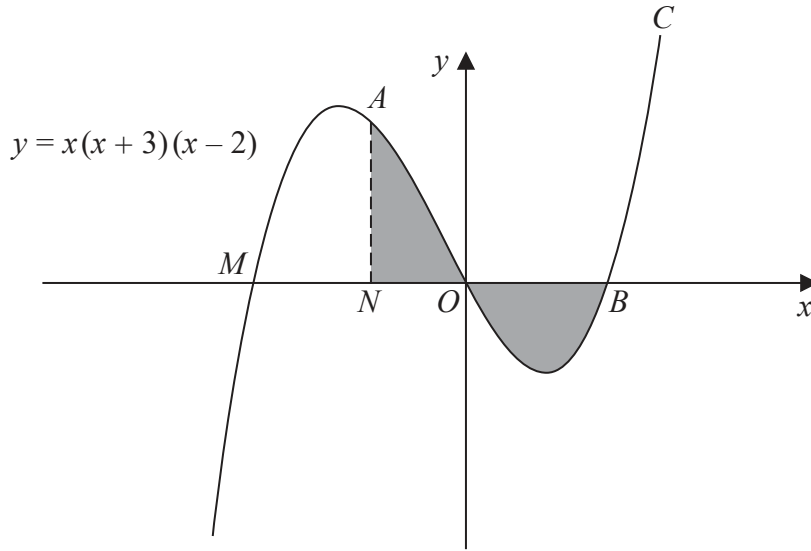


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a sketch of part of the curve  $C$  with equation  $y = x(x + 3)(x - 2)$

The curve  $C$  crosses the  $x$ -axis at the point  $M$ , the origin and the point  $B$ .

The point  $N$  lies on the  $x$ -axis between  $M$  and  $O$ .

The point  $A$  lies on  $C$  such that  $AN$  is parallel to the  $y$ -axis.

The area of the shaded region bounded by the curve and  $OB$  is numerically equal to the area of the shaded region bounded by the curve,  $ON$  and  $NA$ .

Given that the coordinates of  $N$  are  $(n, 0)$ ,

(b) use algebraic integration to show that  $n$  satisfies the equation

$$(x - 2)^2(3x^2 + 16x + 16) = 0 \tag{7}$$

(c) Hence find the exact coordinates of  $A$ .

(5)

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 9 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 25 horizontal dotted lines.



P 7 1 6 6 1 A 0 2 7 3 2

**Question 9 continued**

Area with horizontal dotted lines for writing.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 9 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area with horizontal dotted lines for writing.

**(Total for Question 9 is 16 marks)**



P 7 1 6 6 1 A 0 2 9 3 2

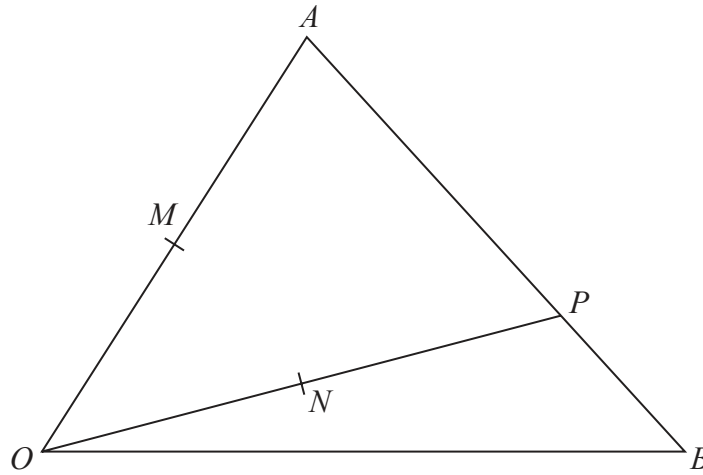


Diagram NOT accurately drawn

Figure 4

Figure 4 shows triangle  $OAB$  in which

$$\vec{OA} = \mathbf{a} \text{ and } \vec{OB} = \mathbf{b}$$

The point  $P$  lies on  $AB$  such that  $AP:PB = 3:1$

The point  $M$  is the midpoint of  $OA$  and the point  $N$  is the midpoint of  $OP$ .

(a) Find, as simplified expressions in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , the vector

- (i)  $\vec{OP}$       (ii)  $\vec{MN}$

(4)

The point  $C$  lies on  $OB$  such that  $ANC$  is a straight line.

(b) Using a vector method, find the vector  $\vec{OC}$  as a simplified expression in terms of  $\mathbf{b}$

(6)

Given that  $\frac{\text{area of quadrilateral } AMNP}{\text{area of triangle } OAB} = K$

(c) find the exact value of  $K$

(4)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 10 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 25 horizontal dotted lines.



P 7 1 6 6 1 A 0 3 1 3 2

